Lecturered By Dr. H. K. Yadav. Deptt. of Mathematics
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B. Sc-part-I(H). paper-I.

Q.1. Define Normal subgroup and the centre Z of group G is a normal subgroup of G.

Soln ~~We know that the definition of the centre of Z, we have~~

~~$Z = \{x \in G : xz = zx, \forall x \in G\}$.~~

Soln. Normal Subgroup:→
Let H be a subgroup of an abelian group G, then

$$Hx = xH, \forall x \in G$$

Sometimes, it is possible that G is a non-abelian group possesses a subgroup H such that $Hx = xH$, $\forall x \in G$. Such subgroups whose left and right cosets coincide are called normal subgroup.

Otherwise, we define normal subgroup: A subgroup H of a group G is said to be a normal subgroup of G if for every $x \in G$ and for every $h \in H$

$$xhx^{-1} \in H.$$

We know that the definition of the centre of Z, we have

$$Z = \{x \in G : xz = zx, \forall x \in G\}.$$

We have proved that Z is a subgroup of G. We have let N be any subgroup of an abelian group G, then

We have, $Hx = xN$, $\forall x \in G$
Hence, N is normal subgroup of G.
Now, to show Z is normal in G.
For all $x \in G$ and $z \in Z$, we have

$$xzx^{-1} = zxx^{-1} \quad [\because xz = zx]$$
$$= ze = z \in Z$$

$$\Rightarrow xzx^{-1} \in Z$$

Hence, Z is normal subgroup of G.

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Q.2. If G is a group and H is a subgroup of index 2 in G, show that H is a normal subgroup of G.

Soln: Since index of H in G is 2, there are two distinct left and right cosets of H in G. Also we have $H = eH = He$, so, H is itself a left as well as right cosets of H in G.

Let $a \in G$. If $a \in H$, then

$$aH = Ha = H$$

Again, for an $a \in G$, $a \notin H$, the left cosets aH is different from H and likewise the right coset Ha is different from H. Since there are only two distinct left and right cosets of H in G, the decomposition of G into left cosets with respect to H consists of H and aH for an $a \in G$, $a \notin H$, therefore

$$G = H \cup aH$$

Also, decomposing G into right cosets with respect to H, we have

$$G = H \cup Ha$$

Now, the left cosets H and aH as well as the right cosets H and Ha have no element in common. it follows from the above relation, we have

$$Ha = aH$$

Now, since a is arbitrary, therefore, every left cosets of H is also a right coset of H.

Hence, H is a normal subgroup of G.

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